MICHIGAN TEST FOR TEACHER CERTIFICATION (MTTC)

TEST OBJECTIVES
FIELD 089: MATHEMATICS (ELEMENTARY)

Subarea	Approximate Percentage of Questions on Test
Mathematical Processes and Number Concepts	28%
Patterns, Algebraic Relationships, and Functions	28%
Measurement and Geometry	22%
Data Analysis, Statistics, Probability, and Discrete Mathematics	22%

The appropriate use of technology (e.g., calculators, computers) is integral to the exploration of concepts, skills, and applications in all areas of mathematics. Although technology is mentioned in some test objectives but not in others, the teacher candidate should be aware of the uses and applications of technology across the range of mathematics topics.

I. MATHEMATICAL PROCESSES AND NUMBER CONCEPTS

001 Understand principles of mathematical reasoning and techniques for communicating mathematical ideas.

Includes analyzing the nature and purpose of axiomatic systems (e.g., understanding the relationships among theorems, postulates, definitions, and undefined terms); using inductive and deductive logic to develop and validate conjectures; applying the laws of deductive logic to draw valid conclusions; developing counterexamples to a conjecture; developing and evaluating direct and indirect proofs; using appropriate mathematical terminology; translating common language into symbols and vice versa; using a variety of numeric, symbolic, and graphic methods to communicate mathematical ideas and concepts; and making connections among numeric, symbolic, graphic, and verbal representations.

002 Understand problem-solving strategies, connections among different mathematical ideas, and the use of mathematical modeling to solve real-world problems.

Includes devising, carrying out, and evaluating a problem-solving plan; evaluating the reasonableness of a solution; applying a range of strategies (e.g., drawing a diagram, working backwards, creating a simpler problem) to solve problems; analyzing problems that have multiple solutions; selecting an appropriate tool or technology to solve a given problem; recognizing connections among two or more mathematical concepts (e.g., area as a quadratic function); exploring the relationship between geometry and algebra; and applying mathematics across the curriculum and in everyday contexts.

003 Understand and apply concepts of proportional reasoning.

Includes analyzing connections between fraction concepts and ratios and proportions; describing the relationship between proportions and direct and inverse variation; analyzing and applying the relationship between proportions and similar figures; applying connections among proportions, probability, and sampling; analyzing a variety of representations of proportional relationships; and modeling and solving problems involving ratios and proportions.

004 Understand number systems and equivalent ways of representing numbers.

Includes applying place value concepts to numeration systems; identifying characteristics and relationships among natural, whole, integer, rational, irrational, and real numbers; using a variety of equivalent representations of numbers (e.g., $\frac{1}{2} = 0.5 = 50\% = \sqrt{\frac{1}{4}}$); applying properties of number operations (e.g., commutative, distributive); applying order relations to numbers; using set operations (e.g., union, intersection, complement); and using manipulatives, verbal expressions, and geometric models to represent numbers.

005 Understand number theory and operations on number systems.

Includes analyzing properties of prime numbers, factors, multiples, and divisibility; applying number properties to manipulate and simplify algebraic expressions; using scientific notation to compute with very large and very small numbers; comparing and contrasting models of operations across number systems (e.g., using a rectangular array to model multiplication of whole numbers and fractions); using manipulatives, verbal expressions, and geometric models to represent number operations; applying and evaluating mental mathematics and estimation strategies; analyzing standard and nonstandard computational algorithms; and solving a variety of problems using number operations.

II. PATTERNS, ALGEBRAIC RELATIONSHIPS, AND FUNCTIONS

006 Describe, analyze, and generalize mathematical patterns.

Includes recognizing and extending numerical and geometric patterns; constructing, representing, and recording patterns using charts, tables, graphs, and matrices; exploring and describing symmetric and spatial patterns (e.g., fractals, tessellations); analyzing and generalizing sequences, series, and recursive patterns; and using patterns to make inferences, predictions, and decisions.

007 Use variables and symbolic expressions to describe and analyze patterns of change and functional relationships.

Includes representing situations using variables and expressions; exploring patterns of change characteristic of families of functions (e.g., linear, quadratic, exponential); translating among verbal, graphic, tabular, and symbolic representations of functions; distinguishing between relations and functions; analyzing functions in terms of range, domain, and intercepts; using piecewise functions; analyzing the relationship among the graphs of $f(x)$ and transformations [e.g., $f(x \pm c)$, $f(x) \pm c$, $cf(x)$, $\frac{1}{f(x)}$]; and using graphing calculators and utilities to analyze properties of functions.

008 Understand properties and applications of linear functions, and solve related equations and inequalities.

Includes describing properties of slope and intercepts; analyzing the relationship between a linear equation and its graph; determining the equation of a line in a variety of situations; modeling problems using linear equations and inequalities; and solving linear systems using a variety of methods (e.g., using substitution, using graphs, using matrices).

009 Understand properties and applications of quadratic functions, and solve related equations and inequalities.

Includes solving quadratic equations, inequalities, and systems using a variety of methods (e.g., graphical, analytical); exploring the zeros, turning point (vertex), and symmetry of a quadratic function; analyzing how changing the coefficients of a quadratic function changes its graph; and using quadratic functions to model and solve problems, including maximum and minimum problems.

010 Understand properties and applications of nonlinear functions and the conceptual foundations of calculus.

Includes using exponential functions to model and solve real-world problems; recognizing the relationship between inverse variation and rational functions; exploring the properties and graphs of polynomial, rational, radical, exponential, logarithmic, and trigonometric (i.e., sine, cosine, tangent) functions; using graphing calculators to solve systems of equations involving these functions; analyzing the relationships among the graph, slope of the secant line, and the derivative of a function; recognizing the relationship between the area under a curve and integration; and describing how calculus is used to solve problems involving dynamic change.

III. MEASUREMENT AND GEOMETRY

011 Understand attributes of measurement and measuring units.

Includes selecting appropriate units (standard and nonstandard) to estimate and record measurements of angle (degree and radian), length, area, volume, mass, temperature, and time; identifying tools for performing measurements; converting measurements within measurement systems; analyzing how changes in the measurement of one attribute relate to changes in others; using dimensional analysis to solve problems; solving problems involving density, pressure, rates of change, and other derived units; and evaluating precision, accuracy, measurement errors, and percent error.

012 Apply measurement principles to analyze the spatial characteristics of two- and three-dimensional shapes.

Includes deriving and applying formulas for the perimeter, area, surface area, or volume of two- and three-dimensional composite figures; exploring scale factors for the area and volume of similar figures; applying right triangle trigonometry and the Pythagorean theorem to solve problems (e.g., problems involving indirect measurements); interpreting three-dimensional drawings of objects; and analyzing cross sections and nets of three-dimensional figures.

013 Apply geometric principles of points, lines, angles, planes, congruence, and similarity to analyze the formal characteristics of two- and three-dimensional shapes.

Includes determining necessary and sufficient conditions for the existence of a particular shape; applying properties of parallel and perpendicular lines and angles to analyze shapes; comparing and analyzing shapes and formally establishing the relationships among them (e.g., congruence, similarity); using geometric principles to prove theorems; applying properties of two-dimensional shapes to analyze three-dimensional shapes; and recognizing the uses of dynamic geometry software in making conjectures and investigating properties of shapes.

014 Apply properties of geometric transformations and coordinate geometry to describe geometric objects in two and three dimensions.

Includes analyzing figures in terms of translations, reflections, rotations, dilations, and contractions; applying transformations to explore the concepts of congruence and similarity; using transformations to characterize the symmetry of an object; locating objects in terms of their position using rectangular coordinate systems; locating and describing the locus of points that satisfy a given condition; and applying concepts of slope, distance, midpoint, and parallel and perpendicular lines to determine the geometric and algebraic properties of figures in the coordinate plane.

IV. DATA ANALYSIS, STATISTICS, PROBABILITY, AND DISCRETE MATHEMATICS

015 Understand methods of organizing, displaying, analyzing, and interpreting data.

Includes organizing data using tables and spreadsheets; creating a variety of charts to display data (e.g., pie charts, box plots, stem and leaf plots, scatter plots, frequency histograms); evaluating the source, organization, and presentation of data; applying and interpreting measures of central tendency (e.g., mean, median, mode) and spread (e.g., range, standard deviation); analyzing the effects of data transformations on measures of central tendency and spread; using appropriate technology to analyze and manipulate data; and evaluating the validity of statistical arguments.

016 Understand methods of collecting data and making predictions and inferences based on data.

Includes applying appropriate techniques for collecting data; analyzing factors that may affect the validity of a survey, including bias; using simulations and sampling to test inferences; applying principles of interpolation and extrapolation; analyzing linear regression lines and correlation coefficients; analyzing the relationship between sample size and width of confidence interval; and employing confidence intervals in making predictions and inferences based on data.

017 Understand the theory of probability and probability distributions.

Includes enumerating the sample space of an event; determining simple and compound probabilities; determining conditional probabilities; finding the probability of dependent and independent events; calculating expected values; using simulations and sampling to determine experimental probabilities; solving problems using geometric probability (e.g., ratio of two areas); applying probability distributions (e.g., binomial, normal) to solve problems; and modeling and solving real-world problems using probability concepts.

018 Understand principles of discrete mathematics.

Includes solving counting problems using permutations and combinations; using sets and set relations to represent algebraic and geometric concepts; using finite graphs and trees to model problem situations; employing recursion and iteration methods to model problems; describing and analyzing efficient algorithms to accomplish a task or solve a problem in a variety of contexts (e.g., practical and computer-related situations); and using linear programming to model and solve problems.