Michigan

TEST FOR TEACHER CERTIFICATION

STUDY GUIDE

089 Mathematics (Elementary)

MI-SG-FLD089-07

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PART 1: General Information About the MTTC Program and Test Preparation

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General Information About the MTTC Program and Test Preparation

PART 2: Test Objectives and Sample Test Questions

INTRODUCTION

This section includes a list of the test objectives, immediately followed by sample test questions and an answer key for the field covered by this study guide.

Test Objectives

As noted, the test objectives are broad, conceptual statements that reflect the knowledge, skills, and understanding an entry-level teacher needs in order to teach effectively in a Michigan classroom. Each field's list of test objectives represents the **only** source of information about what a specific test will cover and, therefore, should be studied carefully.

The test objectives are organized into groups known as "subareas." These subareas define the major content areas of the test. You will find a list of subareas at the beginning of the test objective list. The percentages shown in the list of subareas indicate the approximate weighting of the subareas on the test.

Sample Multiple-Choice Test Questions

The sample multiple-choice test questions included in this section are designed to give the test-taker an introduction to the nature of the test questions included on the MTTC test for each field. The sample test questions represent the various types of test questions you may expect to see on an actual test; however, they are **not** designed to provide diagnostic information to help you identify specific areas of individual strengths and weaknesses or predict your performance on the test as a whole. Use the answer key that follows the sample test questions to check your answers.

To help you identify which test objective is being assessed, the objective statement to which the question corresponds is listed in the answer key. When you are finished with the sample test questions, you may wish to go back and review the entire list of test objectives and descriptive statements once again.

Mathematics (Elementary) (89) Field-Specific Information

Approved Graphing Calculators. Examinees taking the Mathematics (Elementary) or Mathematics (Secondary) test must bring their own graphing calculator but may **not** bring a calculator manual. Graphing calculators will not be provided at the test session. Only the brand and models listed below may be used. **Note that the list of approved graphing calculators for paper-based testing is different than the list for computer-based testing.** Make sure to review the appropriate list for your test. Approved calculator brands and models are subject to change; if there is a change, examinees will be notified. Test administration staff will clear the memory of your calculator both before and after testing. Therefore, be sure to back up the memory on your calculator, including applications, to an external device before arriving at the test center. The list of approved calculators can be found on the Mathematics (Elementary) page at www.mttc.nesinc.com.

TEST OBJECTIVES

Calculators for the Mathematics Test

You may bring your own graphing calculator to the test administration. However, only the brands and models listed on the MTTC website may be used at the administration.

The approved calculator brands and models are subject to change. If there is a change, examinees will be notified.

If you bring your own calculator, test administration staff will clear the memory of your calculator both before and after the test. Be sure that you back up the memory on your calculator, including applications, before arriving at the test site.

Subarea	Approximate Percentage of Questions on Test
Mathematical Processes and Number Concepts	28%
Patterns, Algebraic Relationships, and Functions	28%
Measurement and Geometry	22%
Data Analysis, Statistics, Probability, and Discrete Mathematics	22%

The appropriate use of technology (e.g., calculators, computers) is integral to the exploration of concepts, skills, and applications in all areas of mathematics. Although technology is mentioned in some test objectives but not in others, the teacher candidate should be aware of the uses and applications of technology across the range of mathematics topics.

MATHEMATICAL PROCESSES AND NUMBER CONCEPTS

Understand principles of mathematical reasoning and techniques for communicating mathematical ideas.

Includes analyzing the nature and purpose of axiomatic systems (e.g., understanding the relationships among theorems, postulates, definitions, and undefined terms); using inductive and deductive logic to develop and validate conjectures; applying the laws of deductive logic to draw valid conclusions; developing counterexamples to a conjecture; developing and evaluating direct and indirect proofs; using appropriate mathematical terminology; translating common language into symbols and vice versa; using a variety of numeric, symbolic, and graphic methods to communicate mathematical ideas and concepts; and making connections among numeric, symbolic, graphic, and verbal representations.

Understand problem-solving strategies, connections among different mathematical ideas, and the use of mathematical modeling to solve real-world problems.

Includes devising, carrying out, and evaluating a problem-solving plan; evaluating the reasonableness of a solution; applying a range of strategies (e.g., drawing a diagram, working backwards, creating a simpler problem) to solve problems; analyzing problems that have multiple solutions; selecting an appropriate tool or technology to solve a given problem; recognizing connections among two or more mathematical concepts (e.g., area as a quadratic function); exploring the relationship between geometry and algebra; and applying mathematics across the curriculum and in everyday contexts.

Understand and apply concepts of proportional reasoning.

Includes analyzing connections between fraction concepts and ratios and proportions; describing the relationship between proportions and direct and inverse variation; analyzing and applying the relationship between proportions and similar figures; applying connections among proportions, probability, and sampling; analyzing a variety of representations of proportional relationships; and modeling and solving problems involving ratios and proportions.

Understand number systems and equivalent ways of representing numbers.

Includes applying place value concepts to numeration systems; identifying characteristics and relationships among natural, whole, integer, rational, irrational, and real numbers; using a variety of equivalent representations of numbers (e.g., $\frac{1}{2} = 0.5 = 50\% = \sqrt{\frac{1}{4}}$); applying properties of number operations (e.g., commutative, distributive); applying order relations to numbers; using set operations (e.g., union, intersection, complement); and using manipulatives, verbal expressions, and geometric models to represent numbers.

Understand number theory and operations on number systems.

Includes analyzing properties of prime numbers, factors, multiples, and divisibility; applying number properties to manipulate and simplify algebraic expressions; using scientific notation to compute with very large and very small numbers; comparing and contrasting models of operations across number systems (e.g., using a rectangular array to model multiplication of whole numbers and fractions); using manipulatives, verbal expressions, and geometric models to represent number operations; applying and evaluating mental mathematics and estimation strategies; analyzing standard and nonstandard computational algorithms; and solving a variety of problems using number operations.

PATTERNS, ALGEBRAIC RELATIONSHIPS, AND FUNCTIONS

Describe, analyze, and generalize mathematical patterns.

Includes recognizing and extending numerical and geometric patterns; constructing, representing, and recording patterns using charts, tables, graphs, and matrices; exploring and describing symmetric and spatial patterns (e.g., fractals, tessellations); analyzing and generalizing sequences, series, and recursive patterns; and using patterns to make inferences, predictions, and decisions.

Use variables and symbolic expressions to describe and analyze patterns of change and functional relationships.

Includes representing situations using variables and expressions; exploring patterns of change characteristic of families of functions (e.g., linear, quadratic, exponential); translating among verbal, graphic, tabular, and symbolic representations of functions; distinguishing between relations and functions; analyzing functions in terms of range, domain, and intercepts; using piecewise functions; analyzing the relationship among the graphs of $f(x)$ and transformations [e.g., $f(x \pm c)$, $f(x) \pm c$, $cf(x)$, $\frac{1}{f(x)}$]; and using graphing calculators and utilities to analyze properties of functions.

Understand properties and applications of linear functions, and solve related equations and inequalities.

Includes describing properties of slope and intercepts; analyzing the relationship between a linear equation and its graph; determining the equation of a line in a variety of situations; modeling problems using linear equations and inequalities; and solving linear systems using a variety of methods (e.g., using substitution, using graphs, using matrices).

Understand properties and applications of quadratic functions, and solve related equations and inequalities.

Includes solving quadratic equations, inequalities, and systems using a variety of methods (e.g., graphical, analytical); exploring the zeros, turning point (vertex), and symmetry of a quadratic function; analyzing how changing the coefficients of a quadratic function changes its graph; and using quadratic functions to model and solve problems, including maximum and minimum problems.

Understand properties and applications of nonlinear functions and the conceptual foundations of calculus.

Includes using exponential functions to model and solve real-world problems; recognizing the relationship between inverse variation and rational functions; exploring the properties and graphs of polynomial, rational, radical, exponential, logarithmic, and trigonometric (i.e., sine, cosine, tangent) functions; using graphing calculators to solve systems of equations involving these functions; analyzing the relationships among the graph, slope of the secant line, and the derivative of a function; recognizing the relationship between the area under a curve and integration; and describing how calculus is used to solve problems involving dynamic change.

MEASUREMENT AND GEOMETRY

Understand attributes of measurement and measuring units.

Includes selecting appropriate units (standard and nonstandard) to estimate and record measurements of angle (degree and radian), length, area, volume, mass, temperature, and time; identifying tools for performing measurements; converting measurements within measurement systems; analyzing how changes in the measurement of one attribute relate to changes in others; using dimensional analysis to solve problems; solving problems involving density, pressure, rates of change, and other derived units; and evaluating precision, accuracy, measurement errors, and percent error.

Apply measurement principles to analyze the spatial characteristics of two- and three-dimensional shapes.

Includes deriving and applying formulas for the perimeter, area, surface area, or volume of two- and three-dimensional composite figures; exploring scale factors for the area and volume of similar figures; applying right triangle trigonometry and the Pythagorean theorem to solve problems (e.g., problems involving indirect measurements); interpreting three-dimensional drawings of objects; and analyzing cross sections and nets of three-dimensional figures.

Apply geometric principles of points, lines, angles, planes, congruence, and similarity to analyze the formal characteristics of two- and three-dimensional shapes.

Includes determining necessary and sufficient conditions for the existence of a particular shape; applying properties of parallel and perpendicular lines and angles to analyze shapes; comparing and analyzing shapes and formally establishing the relationships among them (e.g., congruence, similarity); using geometric principles to prove theorems; applying properties of two-dimensional shapes to analyze three-dimensional shapes; and recognizing the uses of dynamic geometry software in making conjectures and investigating properties of shapes.

Apply properties of geometric transformations and coordinate geometry to describe geometric objects in two and three dimensions.

Includes analyzing figures in terms of translations, reflections, rotations, dilations, and contractions; applying transformations to explore the concepts of congruence and similarity; using transformations to characterize the symmetry of an object; locating objects in terms of their position using rectangular coordinate systems; locating and describing the locus of points that satisfy a given condition; and applying concepts of slope, distance, midpoint, and parallel and perpendicular lines to determine the geometric and algebraic properties of figures in the coordinate plane.

DATA ANALYSIS, STATISTICS, PROBABILITY, AND DISCRETE MATHEMATICS

Understand methods of organizing, displaying, analyzing, and interpreting data.

Includes organizing data using tables and spreadsheets; creating a variety of charts to display data (e.g., pie charts, box plots, stem and leaf plots, scatter plots, frequency histograms); evaluating the source, organization, and presentation of data; applying and interpreting measures of central tendency (e.g., mean, median, mode) and spread (e.g., range, standard deviation); analyzing the effects of data transformations on measures of central tendency and spread; using appropriate technology to analyze and manipulate data; and evaluating the validity of statistical arguments.

Understand methods of collecting data and making predictions and inferences based on data.

Includes applying appropriate techniques for collecting data; analyzing factors that may affect the validity of a survey, including bias; using simulations and sampling to test inferences; applying principles of interpolation and extrapolation; analyzing linear regression lines and correlation coefficients; analyzing the relationship between sample size and width of confidence interval; and employing confidence intervals in making predictions and inferences based on data.

Understand the theory of probability and probability distributions.

Includes enumerating the sample space of an event; determining simple and compound probabilities; determining conditional probabilities; finding the probability of dependent and independent events; calculating expected values; using simulations and sampling to determine experimental probabilities; solving problems using geometric probability (e.g., ratio of two areas); applying probability distributions (e.g., binomial, normal) to solve problems; and modeling and solving real-world problems using probability concepts.

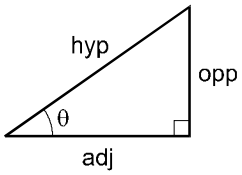
Understand principles of discrete mathematics.

Includes solving counting problems using permutations and combinations; using sets and set relations to represent algebraic and geometric concepts; using finite graphs and trees to model problem situations; employing recursion and iteration methods to model problems; describing and analyzing efficient algorithms to accomplish a task or solve a problem in a variety of contexts (e.g., practical and computer-related situations); and using linear programming to model and solve problems.

SAMPLE MATHEMATICS FORMULAS

Formula	Description
$V = \frac{1}{3}Bh$	Volume of a pyramid
$V = \frac{1}{3}\pi r^2 h$	Volume of a cone
$V = \pi r^2 h$	Volume of a cylinder
$A = 4\pi r^2$	Surface area of a sphere
$V = \frac{4}{3}\pi r^3$	Volume of a sphere
$S_n = \frac{n}{2}[2a + (n-1)d] = n\left(\frac{a + a_n}{2}\right)$	Sum of an arithmetic series
$S_n = \frac{a(1 - r^n)}{1 - r}$	Sum of a geometric series
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance formula
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	Midpoint formula
$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$	Slope
$s = r\theta$	Arc length
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic formula
$y = mx + b$	Slope intercept form of line
$a^2 + b^2 = c^2$	Pythagorean theorem
$D = R \bullet T$	Distance
$\frac{n!}{r!(n-r)!}$	Combinations
$\frac{n!}{(n-r)!}$	Permutations

FORMULAS (continued)

Formula	Description
	
$\sin\theta = \frac{\text{opp}}{\text{hyp}}$	
$\cos\theta = \frac{\text{adj}}{\text{hyp}}$	
$\tan\theta = \frac{\text{opp}}{\text{adj}}$	

SAMPLE MULTIPLE-CHOICE TEST QUESTIONS

Calculators for the Mathematics Test

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1. Sixth-grade students are using a graphing calculator to explore how changing the value of m in an equation of the form $y = mx$ changes the graph of the equation. The students view several graphs and are then asked to make a generalization about how the value of m affects the graph of the equation. This is an example of using:

   A. a counterexample to evaluate a mathematical relationship.

   B. an axiomatic system to generate a mathematical relationship.

   C. inductive reasoning to conjecture a mathematical relationship.

   D. deductive reasoning to prove a mathematical relationship.

2. Students measure the circumference and diameter of a number of circular objects and graph the circumference on the *y*-axis and the diameter on the *x*-axis. The students then draw a line that seems to best fit the data points. How can the students use their graphs to estimate the value of *pi*?

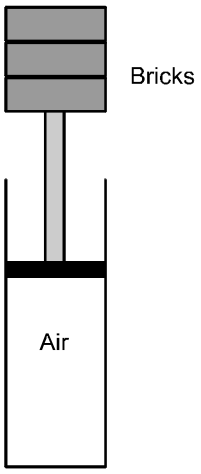
   A. find the length of the line

   B. find the slope of the line

   C. find the *y*-intercept of the line

   D. find the midpoint of the line

3. Use the diagram below to answer the question that follows.



A piston is placed at the top of a cylinder filled with air. As shown in the diagram above, students place a total of three 2 kg bricks, one at a time, on top of the piston and measure the volume of air in the cylinder. Their data are given in the table below.

Mass on Piston (kg)	Volume of Air in Cylinder (cubic centimeters)
2.0	280
4.0	140
6.0	93

What would be the approximate volume of the air in cubic centimeters if students placed two more 2 kg bricks on top of the piston?

A. 35

B. 47

C. 56

D. 70

4. On a calculator, a student starts with 2 and takes the square root four times in succession. This is equivalent to calculating which of the following numbers?

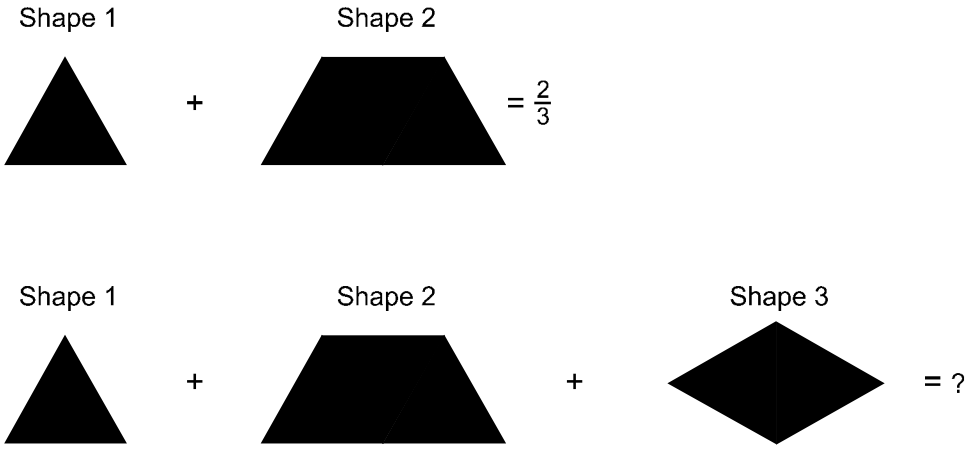
   A. $2^{0.25}$

   B. $\left(\sqrt{2}\right)^{\frac{1}{3}}$

   C. $\left(\sqrt{2}\right)^{\frac{1}{4}}$

   D. $2^{\frac{1}{16}}$

5. A class is using manipulative tiles in the shape of equilateral triangles to explore fractions. One of the students makes up the problem illustrated below.



If Shape 1 and Shape 2 equal $\frac{2}{3}$, what does Shape 1 + Shape 2 + Shape 3 equal?

What is the solution to this problem?

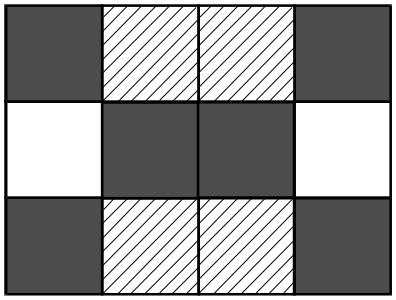
A. $\frac{5}{6}$

B. 1

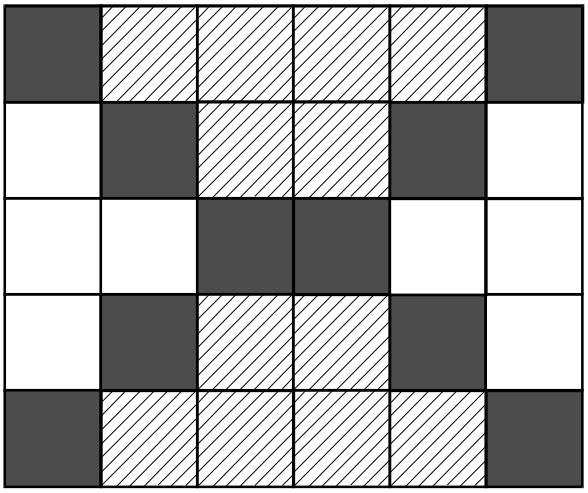
C. $1\frac{1}{3}$

D. $1\frac{1}{2}$

6. The first two elements of a pattern are shown in the diagram below.



1



2

If the pattern continues, how many black boxes will there be in the fifth element of the pattern?

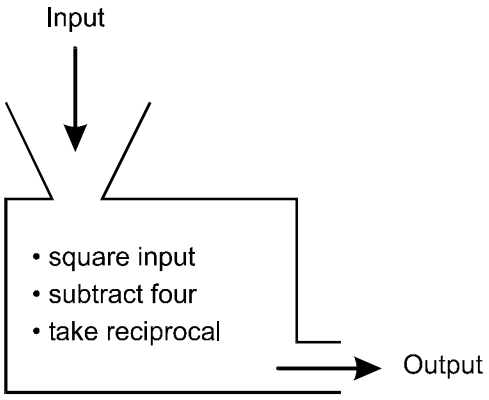
A. 16

B. 18

C. 20

D. 22

7. Use the diagram below to answer the question that follows.



What is the domain of the function machine shown above?

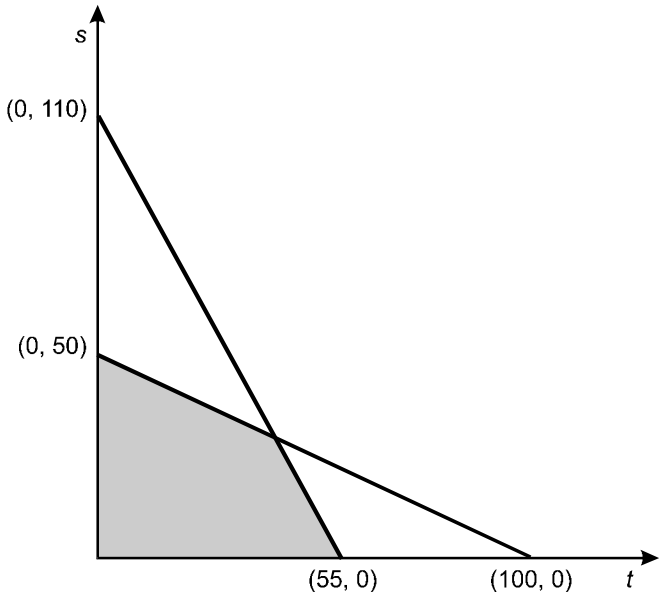
A. all real numbers

B. all real numbers greater than $^{-}4$

C. all real numbers except 2 and $^{-}2$

D. all real numbers except 4 and $^{-}4$

8. Use the diagram below to answer the question that follows.



If $s \geq 0$, and $t \geq 0$, which of the following systems of equations corresponds to the shaded portion of the graph?

A. $t + 2s \leq 50$
$2t + s \leq 55$

B. $2t + s \leq 50$
$t + 2s \leq 55$

C. $2t + s \leq 100$
$t + 2s \leq 110$

D. $t + 2s \leq 100$
$2t + s \leq 110$

9. A quadratic function h(x) has zeros at 4 and $^{-}3$ and a y-intercept of $^{-}12$. The function h(x) is translated $^{-}3$ units on the x-axis. Which of the following equations represents g(x), the transformed h(x)?

   A. $g(x) = x^2 - 5x$

   B. $g(x) = x^2 + 7x$

   C. $g(x) = x^2 - x - 15$

   D. $g(x) = x^2 + 5x - 6$

10. Use the table below to answer the question that follows.

Year	Population
0	50,000
1	51,000
2	52,020
n	?

The table gives the population of a town over a three-year period. If the population continues to increase at the same rate per year, which of the following equations could be used to predict the population, P(n), in the nth year?

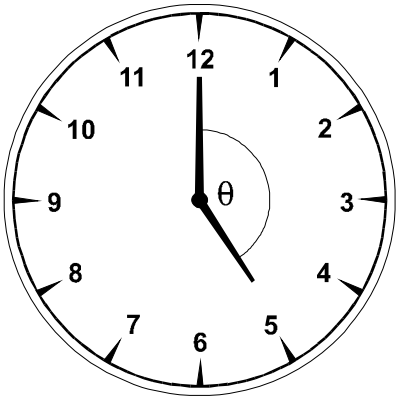
A. $P(n) = 50{,}000 + 500n$

B. $P(n) = 50{,}000(0.02)^n$

C. $P(n) = 50{,}000 + 1{,}000n$

D. $P(n) = 50{,}000(1.02)^n$

11. Use the diagram below to answer the question that follows.



What is the measure of the angle θ between the two hands of an analog clock when the clock reads 5:00?

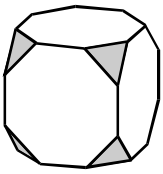
A. 120°

B. 150°

C. 160°

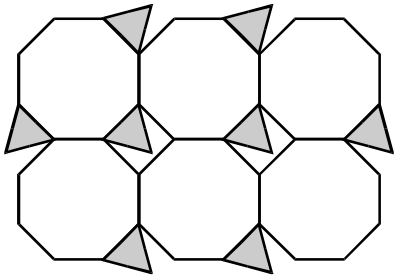
D. 175°

12. Use the diagram below to answer the question that follows.

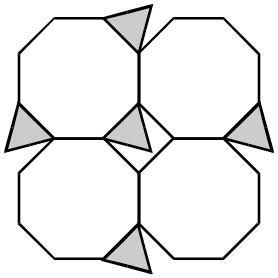


A truncated cube is a polyhedron formed by cutting the corners off a cube. Which of the following nets represents the truncated cube shown above?

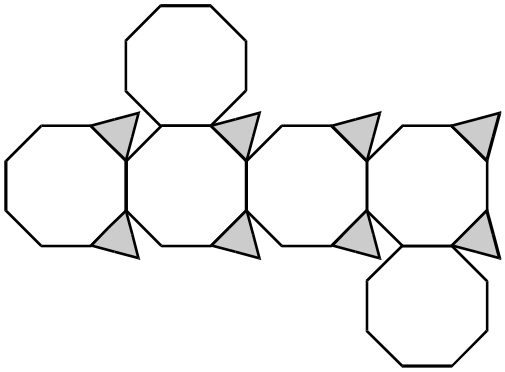
A.



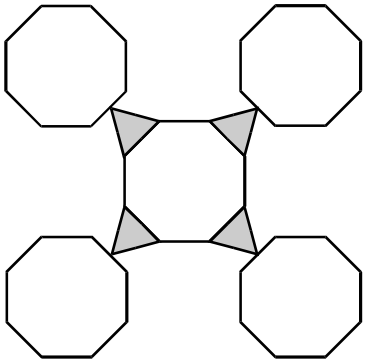
B.



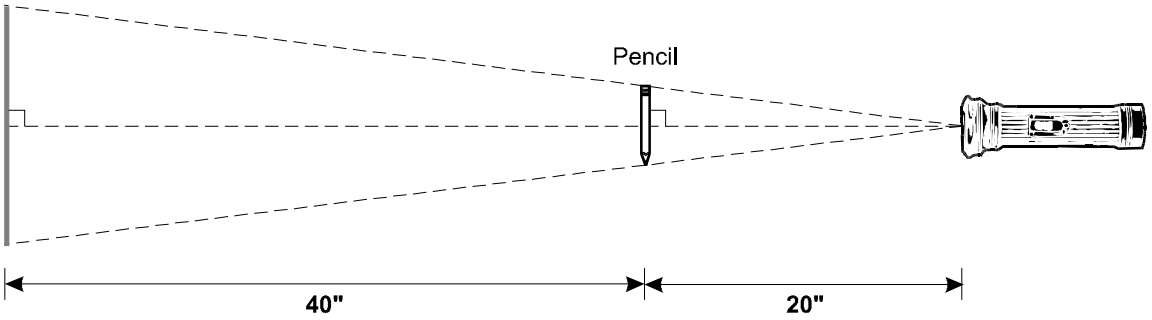
C.



D.



13. Use the diagram below to answer the question that follows.



A pencil 5 inches long is held between a wall and a flashlight creating a shadow on the wall as shown above. What is the length of the pencil's shadow in inches?

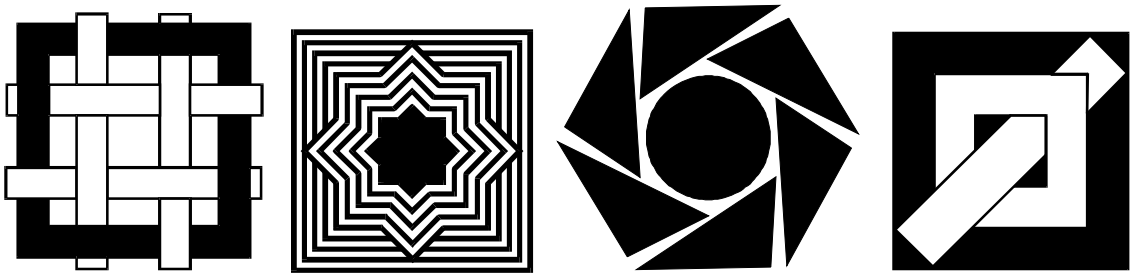
A. 10

B. 12

C. 15

D. 20

14. Use the figures below to answer the question that follows.



Which of the figures above has both reflective and rotational symmetry?

A. Figure A

B. Figure B

C. Figure C

D. Figure D

15. Use the table below to answer the question that follows.

Vertex	Coordinates
E	(0, 2, 2)
F	(6, 2, 2)
G	(6, 2, 0)
H	(6, 0, 0)

The *x*-*y*-*z*-coordinates of four of a rectangular solid's vertices are given in the table. What is the surface area of this solid?

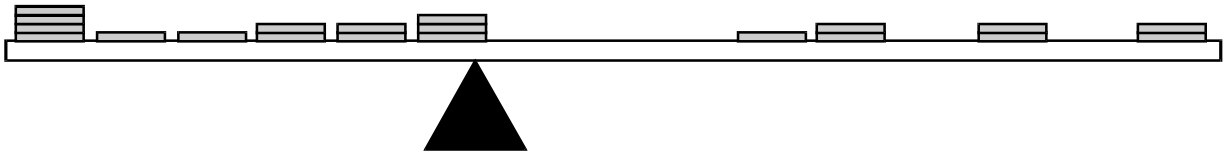
A. 24

B. 40

C. 56

D. 72

16. Use the diagram below to answer the question that follows.



In a unit on measures of central tendency, a middle school math teacher places 20 pennies on top of a straight edge. She then balances the straight edge on a triangular block. Which of the following measures of central tendency is most analogous to the point at which the straight edge balances?

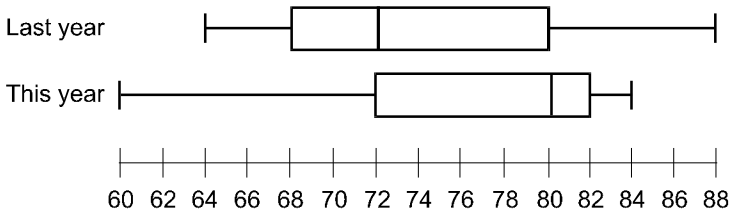
A. variance

B. mean

C. mode

D. median

17. Use the diagram below to answer the question that follows.



The plots above display state assessment results. Based on these plots, which of the following is an accurate statement to report to parents?

A. Unfortunately, our scores this year were not as high as last year. This year a higher percentage of students than last year had raw scores below 72.

B. Unfortunately, our scores this year were not as high as last year. Last year a higher percentage of students scored above 82.

C. Fortunately, our scores this year improved. The percentage of students in the interquartile range increased this year.

D. Fortunately, our scores this year improved. Seventy-five percent of our students scored at or above last year's median.

18. A student is using a computer program to graph the equation of a line in the form $y = mx + b$. If both m and b are elements of the set {1, 2, 3}, how many distinct lines can be drawn?

A. 3

B. 6

C. 8

D. 9

19. A newspaper is planning to conduct a poll to determine what percentage of the registered voters in a community supports the proposed school budget for the next year. In designing the poll, the newspaper's staff should recognize that increasing the number of people included in the survey will:

    A. strengthen the null hypothesis.

    B. narrow the confidence interval reported.

    C. increase the statistical significance of the results.

    D. reduce the randomness of the sample.

20. Friendly Farms, Inc., has several farms. Over the years, it has been determined that the probability of sufficient rain during the maximum growth period is 25%. When there is not enough rain, Friendly Farms needs to supplement with irrigation. The profit when there is sufficient rain during the maximum growth period is $4000 per acre. If it does not rain during this period, the profit is only $1600 per acre. Using this information, what profit per acre, on average, can Friendly Farms expect to make?

    A. $1000

    B. $2200

    C. $2800

    D. $3400

ANSWER KEY FOR THE SAMPLE MULTIPLE-CHOICE TEST QUESTIONS

Item Number	Correct Response	Objective
1.	**C**	Understand principles of mathematical reasoning and techniques for communicating mathematical ideas.
2.	**B**	Understand problem-solving strategies, connections among different mathematical ideas, and the use of mathematical modeling to solve real-world problems.
3.	**C**	Understand and apply concepts of proportional reasoning.
4.	**D**	Understand number systems and equivalent ways of representing numbers.
5.	**B**	Understand number theory and operations on number systems.
6.	**D**	Describe, analyze, and generalize mathematical patterns.
7.	**C**	Use variables and symbolic expressions to describe and analyze patterns of change and functional relationships.
8.	**D**	Understand properties and applications of linear functions, and solve related equations and inequalities.
9.	**D**	Understand properties and applications of quadratic functions, and solve related equations and inequalities.
10.	**D**	Understand properties and applications of nonlinear functions and the conceptual foundations of calculus.
11.	**B**	Understand attributes of measurement and measuring units.
12.	**C**	Apply measurement principles to analyze the spatial characteristics of two- and three-dimensional shapes.
13.	**C**	Apply geometric principles of points, lines, angles, planes, congruence, and similarity to analyze the formal characteristics of two- and three-dimensional shapes.
14.	**B**	Apply properties of geometric transformations and coordinate geometry to describe geometric objects in two and three dimensions.
15.	**C**	Apply properties of geometric transformations and coordinate geometry to describe geometric objects in two and three dimensions.
16.	**B**	Understand methods of organizing, displaying, analyzing, and interpreting data.
17.	**D**	Understand methods of organizing, displaying, analyzing, and interpreting data.
18.	**D**	Understand principles of discrete mathematics.
19.	**B**	Understand methods of collecting data and making predictions and inferences based on data.
20.	**B**	Understand the theory of probability and probability distributions.