Michigan

TEST FOR TEACHER CERTIFICATION

STUDY GUIDE

096 Professional Readiness Examination (Mathematics)

Effective after October 1, 2013

MI-SG-FLD096M-02

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The first section of the study guide is available in a separate PDF file. Click the link below to view or print this section.

General Information About the MTTC Program and Test Preparation

PART 2: Test Objectives and Sample Test Questions

ABOUT THE PROFESSIONAL READINESS EXAMINATION

The Michigan Test for Teacher Certification (MTTC) Professional Readiness Examination is designed to measure communication and mathematical skills. The Professional Readiness Examination consists of approximately 129 multiple-choice questions and two written constructed-response assignments and assesses the following three broad areas of knowledge and skills:

- Reading
- Mathematics
- Writing

You will receive a separate score for each subtest of the test (i.e., Reading, Mathematics, Writing). A passing score is required for each subtest in order to pass the Professional Readiness Examination. Examinees who do not pass one or more subtests of the Professional Readiness Examination may retake only those subtests. Once a subtest is passed, you do not need to retake that subtest. If you have previously passed one or more Basic Skills (096) subtests, your passing status for each subtest you passed will be accepted, and you will only need to take and pass any subtest(s) you have not yet passed. Refer to the MTTC Web site at **www.mttc.nesinc.com**, for more information about retaking subtests of the Professional Readiness Examination.

Test Objectives

The test objectives are broad, conceptual statements that reflect the knowledge, skills, and understanding important for a candidate to have before beginning the student teaching requirement in Michigan. These test objectives represent the **only** source of information about what the test will cover and therefore should be studied carefully.

The test objectives are organized into groups known as "subareas." These subareas define the major content areas of the test. You will find a list of subareas at the beginning of the test objective list.

Sample Multiple-Choice Test Questions

The sample multiple-choice test questions included in this section are designed to give the test-taker an introduction to the nature of the test questions included on the MTTC Professional Readiness Examination. The sample test questions represent the various types of test questions you may expect to see on an actual test; however, they are **not** designed to provide diagnostic information to help you identify specific areas of individual strengths and weaknesses or predict your performance on the test as a whole. Use the answer key that follows the sample test questions to check your answers.

Reading

Many test questions in the Reading Subtest are based on reading passages. Consider the strategies below and choose the one that works best for you.

A first strategy is to read the passage thoroughly and carefully and then answer each question, referring to the passage only as needed. A second strategy is to read the questions first, gaining an idea of what is sought in them, and then read the passage with the questions in mind. A third strategy is to scan the passage very quickly to gain an overview of its content, and then answer each question by referring back to the passage for the specific answer. Any of these strategies is appropriate. However, you should not answer the questions on the basis of your own opinions but rather on the basis of the ideas and opinions expressed in the passage.

In the Reading Subtest, the order of information requested by the questions does not necessarily correspond to the order of information in a given selection. Certain words and phrases within some selections have been underlined for testing purposes only; underlined words do not reflect emphasis intended by the writers.

Mathematics

In the Mathematics Subtest, a set of mathematical definitions and formulas is provided that may help you perform the calculations on the test. A sample of these definitions and formulas is provided following the sample test directions. A four-function calculator will be provided for use with this test. For computer-based testing, an on-screen calculator will be available for your use during the test. For paper-based testing, a four-function calculator will be provided with your test materials.

Writing

The Writing Subtest consists of forty-two multiple-choice questions and two written constructed-response assignments. This study guide provides sample multiple-choice questions, writing assignment performance characteristics, a scoring scale, practice constructed-response writing assignments, and sample responses to the constructed-response writing assignments.

For computer-based testing, you may use the provided note booklet to plan your responses, but your final responses must be entered into the computer. For paper-based testing, you may use blank space in the test booklet to plan your responses, but your final responses must be written in the written response booklet.

TEST OBJECTIVES

Subarea	Approximate Percentage of Questions on Test
Quantitative Literacy and Logic	20%
Algebra and Functions	30%
Geometry and Trigonometry	30%
Statistics and Probability	20%

VII. QUANTITATIVE LITERACY AND LOGIC

007 Understand numbers, numbers systems, and the relationships between them.

Includes:

- demonstrating knowledge of properties of integers, rational numbers, and real numbers (e.g., additive and multiplicative identities and inverses; associative, commutative, and distributive properties)
- using mathematical symbols in context to represent quantitative relationships and situations (e.g., inequalities, absolute values, vectors, logarithms, exponents, complex numbers)

008 Apply skills in performing calculations, using algorithms, and estimating solutions.

Includes:

- demonstrating knowledge of sequences and iteration (e.g., terms of arithmetic, geometric, and other simple sequences; sums of finite arithmetic and geometric sequences; the use of iterative processes in computing compound interest)
- applying skills related to measurement units and scales (e.g., units of measurement, conversion of units, calculations involving measurements and units)
- solving problems involving error analysis (e.g., significant digits, error tolerance, percent of error, accumulated error, rounding error, truncating)
- solving problems using number sense
- performing calculations involving real and complex numbers

009 Understand mathematical reasoning, logic, and proof.

Includes:

- demonstrating knowledge of inductive and deductive reasoning
- analyzing logical statements and arguments using the rules of logic (e.g., negation, contrapositive, inverse, converse, connectives)
- demonstrating knowledge of mathematical proofs (e.g., use of axioms, postulates, theorems, definitions, counterexamples, contradiction)
- demonstrating knowledge of necessary and sufficient conditions

VIII. ALGEBRA AND FUNCTIONS

010 Understand expressions, equations, and inequalities.

Includes:

- constructing and interpreting algebraic expressions (e.g., verbal descriptions of expressions in symbolic form, symbolic representation of verbal descriptions, evaluation of algebraic expressions)
- solving problems involving manipulation of algebraic expressions (e.g., performing operations, simplifying, factoring, transforming exponential and logarithmic expressions)
- demonstrating knowledge of linear, quadratic, polynomial, and rational equations and inequalities

011 Understand functions, their representations, and their characteristics.

Includes:

- recognizing definitions and representations of functions including piecewise functions and recursive functions (e.g., recognizing functions and their domains and ranges in contextual, symbolic, tabular, and graphical forms; evaluating functions at a value; representing functions in graphs, tables, diagrams, and words)
- solving problems involving operations on functions (e.g., addition, subtraction, multiplication, division, transformations, inverses)
- recognizing functions as applied to real-world situations (e.g., exponential growth and decay, cost function)

012 Understand families of functions and their symbolic and graphical representations.

Includes:

- solving problems involving lines and linear functions
- solving problems involving exponential and logarithmic functions
- solving problems involving quadratic functions
- solving problems involving power functions
- solving problems involving polynomial functions
- solving problems involving rational functions
- solving problems involving trigonometric functions

IX. GEOMETRY AND TRIGONOMETRY

013 Understand basic geometric figures and their properties.

Includes:

- demonstrating knowledge of coordinate geometry
- recognizing and using formulas involving perimeter, area, and volume
- solving problems involving geometry of two-dimensional figures (e.g., triangles, circles, quadrilaterals)
- solving problems involving geometry of three-dimensional figures (e.g., cone, pyramid, sphere)
- solving problems involving triangles and trigonometry

014 Understand the relationships between basic geometric figures.

Includes:

- recognizing the relationships between area formulas of two-dimensional figures
- recognizing the relationships between volume formulas of three-dimensional figures
- recognizing the relationship between two-dimensional and three-dimensional figures (e.g., recognizing three-dimensional figures from two-dimensional views, cross sections of three-dimensional figures, solids formed by revolving two-dimensional figures around lines)
- demonstrating knowledge of congruence and similarity

015 Understand transformations of figures in a plane.

Includes:

- recognizing isometries (i.e., reflections, rotations, translations, and glide reflections)
- recognizing dilations
- recognizing the composition of two or more transformations

X. STATISTICS AND PROBABILITY

016 Understand univariate data and distributions.

Includes:

- demonstrating knowledge of statistical significance, margin of error, and confidence level
- solving problems involving measures of center (i.e., mean, mode, median), weighted average, and variation (i.e., percentile, quartile, interquartile, range, variance, standard deviation)
- demonstrating knowledge of the normal distribution (e.g., shape, relationships between measures of center, percent of population at various standard deviations from the mean, z-scores)

017 Understand bivariate data and relationships.

Includes:

- demonstrating knowledge of scatterplots and their characteristics
- demonstrating knowledge of Pearson's coefficient of correlation
- differentiating between correlation and causation
- demonstrating knowledge of linear regression

018 Understand sampling, surveying, experimental design, and graphical representation.

Includes:

- constructing and interpreting graphic representations of data (e.g., tables, charts, plots, graphs, spreadsheets, histograms, bar graphs)
- demonstrating knowledge of sample statistics and population parameters
- demonstrating knowledge of sources of bias and procedures for reducing and controlling bias
- demonstrating knowledge of observational studies and experimental studies

019 Understand probability models and probability calculation.

Includes:

- demonstrating knowledge of sample spaces in simple situations
- calculating the probability of events in various situations (e.g., mutually exclusive events, independent events, dependent events, compound events, complementary events, conditional events)
- demonstrating knowledge of tree diagrams, formulas for combinations and permutations, Venn diagrams, and other counting techniques

SAMPLE DEFINITIONS AND FORMULAS

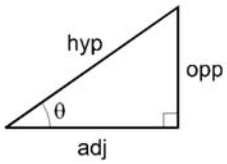
DEFINITIONS

$\approx$ approximately equal to	$\sim$ similar to	$\perp$ perpendicular to
$>$ greater than	$\cong$ congruent to	$\parallel$ parallel to
$<$ less than	π ≈ 3.14	$\overline{AB}$ line segment AB
	$\angle$ angle	$\overleftrightarrow{AB}$ line AB

FORMULAS

Formula	Description
Algebra	
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance formula
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	Midpoint formula
$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$	Slope
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic formula
$y = mx + b$	Slope intercept form of line
$d = r \cdot t$	Distance
$b^x = n$	Exponential
$\log_b n = x$	Logarithm
Statistics and Probability	
$\frac{n!}{r!(n-r)!}$	Combinations
$\frac{n!}{(n-r)!}$	Permutations
$z = \frac{x - \mu}{\sigma}$	*z*-score
$(n)(n-1)(n-2)...(3)(2)(1)$	$n!$

Trigonometry



$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

Angle (θ)	sin (θ)	cos (θ)	tan (θ)
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	undefined

Geometry and Measurement

$C = 2\pi r$	Circumference of a circle
$A = \frac{1}{2}bh$	Area of a triangle
$A = \pi r^2$	Area of a circle
$A = 2\ell h + 2\ell w + 2hw$	Surface Area of a rectangular box
$V = \ell wh$	Volume of a rectangular box
$A =$ *sum of areas of polygonal faces*	Surface Area of a pyramid
$V = \frac{1}{3}Bh$	Volume of a pyramid
$A = \pi rs + \pi r^2$	Surface Area of a cone
$V = \frac{1}{3}\pi r^2 h$	Volume of a cone
$A = 2\pi rh + 2\pi r^2$	Surface Area of a cylinder
$V = \pi r^2 h$	Volume of a cylinder
$A = 4\pi r^2$	Surface area of a sphere
$V = \frac{4}{3}\pi r^3$	Volume of a sphere
$a^2 + b^2 = c^2$	Pythagorean theorem

End of Definitions and Formulas

SAMPLE MULTIPLE-CHOICE TEST QUESTIONS

1. Use the information below to answer the question that follows.

Line 1	$3x + 2 = 11$
Line 2	$3x = 9$

Which of the following properties is applied to the equation in line 1 to obtain the equation in line 2?

A. additive inverse

B. commutative property of addition

C. multiplicative inverse

D. associative property of multiplication

2. Which of the following is equal to $|3 - 7| \bullet 2 - 3^2 + |6 - 9|$?

A. –23

B. –20

C. 1

D. 2

3. What is $\log_4 64 + \log_4 4$?

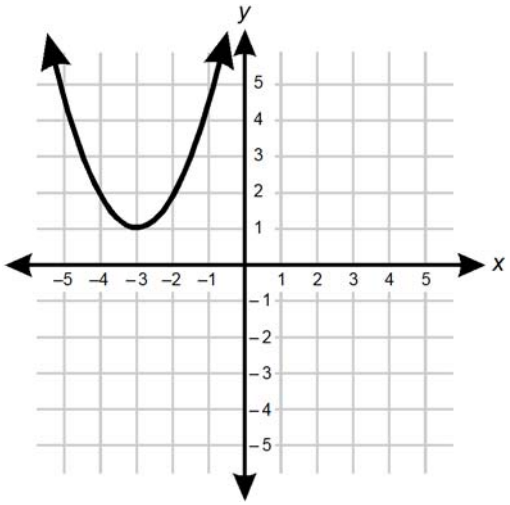
A. 3

B. 4

C. 8

D. 17

4. Use the graph below to answer the question that follows.



Which of the following could be the equation of the curve graphed above?

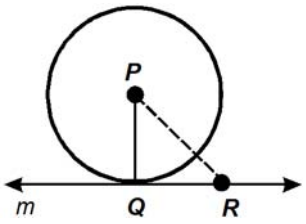
A. $y = x^2 + 6x + 10$

B. $y = x^2 – 6x + 10$

C. $y = x^2 + 6x + 7$

D. $y = x^2 – 6x + 7$

5. Use the diagram below to answer the question that follows.



In the diagram above, line *m* is tangent to circle *P*. For what angle measure of $\angle P$ will $QR = QP$?

A. 30°

B. 45°

C. 60°

D. 90°

6. A rectangle has vertices located at (0, 0), (1, 0), (1, 4), and (0, 4). The rectangle undergoes a dilation with a scale factor of 3. What is the area of the image of the rectangle?

   A. 4

   B. 12

   C. 18

   D. 36

7. A bag contains five marbles numbered from 1 to 5. A marble is taken from the bag, its number is recorded and it is returned to the bag. This is repeated one more time with a second marble. How many elements are found in the sample space for this experiment?

   A. 5

   B. 10

   C. 25

   D. 32

ANSWER KEY FOR THE SAMPLE MULTIPLE-CHOICE TEST QUESTIONS

Item Number	Correct Response	Objective
1.	**A**	Understand numbers, numbers systems, and the relationships between them.
2.	**D**	Apply skills in performing calculations, using algorithms, and estimating solutions.
3.	**B**	Understand expressions, equations, and inequalities.
4.	**A**	Understand families of functions and their symbolic and graphical representations.
5.	**B**	Understand basic geometric figures and their properties.
6.	**D**	Understand transformations of figures in a plane.
7.	**C**	Understand probability models and probability calculation.